Economic Theory **18**, 753–760 (2001)

Economic Theory

On competition and endogenous firm efficiency*

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Received: March 2, 2000; revised version: September 19, 2000

Summary. Conventional wisdom holds that product market competition disciplines firms into efficiency of operation. However, in a well known paper, Martin (1993) has shown that in a linear Cournot setting (with costs determined first and product market competition taking place in a second stage) the exact opposite obtains — a larger number of firms competing in the market implies lower firm efficiency. The note clarifies further the links between market structure and efficiency. Specifically, it argues why (and how) the result derived by Martin (1993) depends upon the assumptions made regarding the structure of demand and nature of conjectures held by firms as to their rivals' behavior. An illustrative counter-example (with Bertrand behavior and non-linear demand) in which entry increases efficiency is provided as well.

Keywords and Phrases: Competition, Endogenous efficiency, Managerial firms, Entry.

JEL Classification Numbers: F02, F12, F13, F15.

1 Introduction

Conventional wisdom holds that product market competition increases firm efficiency while market power fosters sloth and "X-inefficiency". Several attempts to formalize this argument and establish links between market structure and firm efficiency have now been made.[1] However, in a recent and well known contribu-

* The hospitality, financial and intellectual support of the Center for Research on Economic Development and Policy Reform at Stanford University where this research was completed is gratefully acknowledged.

[1] See, for instance, Hart (1983), Scharfstein (1988), and Willig (1987) which have each examined the implications of greater competition for efficiency and operating slack in firms.

tion that paid explicit attention to agency issues and product market interactions,[2] Martin (1993) demonstrated a result that stands in sharp contrast to the popular intuition on this matter — In a linear Cournot setting, firm costs were shown to be inversely related the number of firms competing in the market.

The note clarifies further the links between market structure and efficiency. Specifically, it argues why (and how) the result derived by Martin (1993) depends upon the assumptions made regarding the structure of demand and nature of conjectures held by firms as to their rivals' behavior. A counter-example illustrates this point: With Bertrand competition and non-linear demand, it is shown here that the opposite result, which confirms the conventional wisdom, can be obtained - An increase in the number of firms competing in the product market results in a reduction in firm costs.

The rest of this note proceeds as follows: Section II outlines the model (following Martin (1993) closely), re-states Martin (1993)'s primary result and argues its sensitivity to assumptions regarding demand and the nature of competition between firms. Section III provides the illustrative counter example and concludes.

2 Corporate governance, product market competition and endogenous firm efficiency

There are assumed to be $n+1$ firms competing in the product market. The owners of each firm are assumed to require managers to run them. In each firm, i, the marginal (and average) cost of production is assumed to be given by,

$$c_i(\epsilon_i) = \alpha + \epsilon_i e^{-L_i}, \qquad i = 1, \dots n+1, \tag{1}$$

where $\alpha > 0$, L_i is the labor of the manager of firm i and ϵ_i is a non negative random variable with continuous density function $f(\epsilon_i)$ and with $0 < \underline{\epsilon} \leq \epsilon_i \leq \bar{\epsilon}$. The manager's utility from working is assumed to be given by $U = \phi_i - \lambda L_i$, where λ denotes the manager's marginal disutility of labor and ϕ_i denotes the fee paid to the manager by the owner. Without loss of generality, the manager's reservation utility is assumed to be zero.

Following Martin (1993), the interaction between the owner and the manager is assumed to proceed along the following lines: First, the manager of the firm observes ϵ_i and makes a (potentially dishonest) utility maximizing report of this to the owner. The owner who can observe neither ϵ_i nor the eventual labor choice of the manager, L_i, indirectly controls the manager's actions by establishing a cost target $c_i(\hat{\epsilon}_i)$ and a fee schedule $\phi_i(\hat{\epsilon})$ that depends upon the value $\hat{\epsilon}_i$ of the random variable that the manager reports to the owner (where the manager is to achieve the specified cost level if he is to be paid any fee at all). The manager

[2] As Willig (1987) notes, one view point holds that the nature of the output market is irrelevant to internal efficiency as long as the market for managerial services, either within a firm, or across firms, is competitive enough to induce efficiency on its own. As in much subsequent work, Willig (1987) included, the present analysis examines settings without any competition between managers.

therefore puts in an effort level L_i so that the cost $c_i(\hat{\epsilon})$ is achieved. Product market competition ensues in the *next* stage.

The revelation principle implies that the owner may focus attention on schedules that induce the manager to report truthfully the random component of costs. It is easily verified (See Martin (1993)) that the fee schedule/cost target pair that accomplishes this is given by:

$$\phi_i(\hat{\epsilon}_i) = \lambda log \frac{\bar{\epsilon}}{c_i(\hat{\epsilon}_i) - \alpha}. \tag{2}$$

Since truth telling is induced by this schedule, the manager's report, $\hat{\epsilon}_i$, is simply equal to true value of ϵ_i drawn by nature (and we therefore drop the hat notation henceforth). This fee schedule (2) can be used to express the principal's optimization problem in terms of costs. The principal of firm i picks a cost target $c_i(\epsilon_i)$ that maximizes expected payoff, taking the costs of other firms as given. Letting $\Pi_i(c_i, c_{-i}, n)$ denote profits *exclusive* of managerial compensation, the owner's maximization problem can be written down as:

$$E_\epsilon(V_i) = \int_{\epsilon_1} \ldots \int_{\epsilon_{n+1}} \Pi_i f(\epsilon_{n+1}) \ldots f(\epsilon_1) \; d\epsilon_{n+1} \ldots d\epsilon_1 - \lambda log \bar{\epsilon} + \lambda \int_{\epsilon_i} log(c_i - \alpha) d\epsilon_i, \tag{3}$$

where E_ϵ denotes that the expectation is being taken with respect to the random vector, $\epsilon = [\epsilon_1, \ldots., \epsilon_{n+1}]$. The first order necessary conditions for the maximization of (3), obtained by differentiating under the integral with respect to own costs and setting the result equal to zero, are given by,

$$E_{\epsilon_{-i}}[\frac{\partial \Pi_i}{\partial c_i}] + \frac{\lambda}{c_i - \alpha} = 0. \tag{4}$$

Note that this holds for all values of ϵ_i in the interval $(\underline{\epsilon}, \bar{\epsilon})$ Thus, the optimal cost target is a constant, independent of the realized value of ϵ_i.[3] Since the optimal cost target of the other firms is a constant, $E_{\epsilon_{-i}}[\frac{\partial \Pi_i}{\partial c_i}]$ is simply equal to $[\frac{\partial \Pi_i}{\partial c_i}]$. Thus, the first order condition of the profit maximization problem can be written,

$$[(\frac{\partial \Pi_i}{\partial c_i})] + \frac{\lambda}{c_i - \alpha} = 0. \tag{5}$$

Noting that $\Pi_i = (P_i - c_i)Q_i$, one may re-write equation (5) as follows:

$$[Q_i] - [(P_i - c_i)\frac{\partial Q_i}{\partial c_i} + Q_i \frac{\partial P_i}{\partial c_i}] = [\frac{\lambda}{c_i - \alpha}]. \tag{6}$$

Equation (6) is central to the point being made in this paper and deserves careful discussion. Note that on the left hand side of the equation there are two terms both of which should be familiar from earlier discussions in the literature (See Mas-Collel, Whinston and Green (1995) and Dixit (1986)). The first term is

[3] On this, see also the paper by Bertoletti and Poletti (1996).

the *direct* effect of cost reductions. By itself, its presence in (6) implies that the greater are sales, the lower is the chosen level of marginal cost. The reasoning for this should be clear. A unit reduction in marginal costs (holding everything else fixed) results in a cost savings of Q_i. Therefore, when larger quantities are sold in equilibrium, there is a greater incentive for the owner to induce lower marginal costs. Indeed, if cost decisions and output decisions were made simultaneously, the direct effect alone would determine the optimal cost level chosen.[4] However, in the present framework, costs are chosen in a prior stage. This two stage aspect of the model is reflected in the second term (well known and central to the literature on strategic pre-commitment to affect future competition), the so-called *indirect* or *strategic* effect. If this term (inside the brackets) is negative, we are within a "strategic-substitutes" context implying that that there is a benefit to greater reduction in costs (beyond what is dictated by the magnitude of the direct effect alone). If, on the other hand, the term (inside the brackets) is positive, we are within a "strategic complements" context implying that there is a benefit to raising costs.[5]

Our interest here, however, is not in the sign and magnitude of the terms on the left hand side of (6), but rather in the *change* in these terms due to the presence of a larger number of firms competing in the market. Consider first the *direct* effect: It is generally the case that per-firm output is not increased as the number of firms increases. Thus, the impact of entry on the direct effect is generally a negative one - greater competition leads to higher costs being chosen in equilibrium. We have, however, the strategic effect to reckon with as well - and the impact of entry on the strategic effect is much more complicated. Expanding the terms in the second bracket on the left hand side of (6) will quickly reveal that the strategic effect depends substantially upon both the nature of the demand curve and the conjectures held by firms as to their rivals' responses.[6] The extent to which these terms change, given an increase in the number of firms operating in a market is difficult to determine in full generality. The overall impact of entry (summing over its impact on the direct effect and the strategic effect) may in turn be positive or negative - depending upon the particular market and firm conjectures under consideration. Greater competition may increase or decrease the endogenously chosen level of firm efficiency.

In Martin's (1993) linear Cournot model, entry, holding costs constant, lowers the first term (sales) while making the second more positive. There, the extent to which the first term is lowered is greater than the extent to which the second term is increased — and we have higher costs chosen in equilibrium. It should

[4] This can be seen by noting that if costs and output decisions were made simultaneously by the firm, the first term on the left hand side of (6), would simply involve the partial of the profit function with respect to costs, i.e., Q_i.

[5] Strategic substitutes implies that a lowering of costs forces less aggressive behavior by rivals while strategic complements implies, conversely, that a lowering of costs forces more aggressive behavior on the part of rivals. For instance, the former obtains in the textbook linear demand-Cournot competition context and the latter in the case with linear demand and Bertrand competition. See Mas-Collel, Whinston and Green (1995) for a detailed discussion.

[6] See Dixit (1986) for a detailed derivation and discussion.

be clear from the preceding discussion, however, that this need not be always obtained. Indeed, the opposite may be shown to be true: Entry may lower costs if the change in the strategic effect overwhelms its impact on the direct effect. It is precisely this that we now proceed to demonstrate in the context of a specific illustrative example with firms holding Bertrand conjectures and with demand taking a particular non-linear form.

3 A counter example

Assume that there are $n+1$, $n \geq 1$, firms producing differentiated products. Aggregate demand faced by any firm is assumed to be representable (locally) by a function of own price P_i and the average of the prices charged by the rest of the firms, Ω_i:

$$Q_i = \frac{\theta\Omega_i - P_i}{P_i^2}, \tag{7}$$

where $\Omega_i = (\frac{1}{n})\sum P_j$, $j = 1 \ldots n+1, j \neq i$ and $\theta > 1$.[7] The equilibrium concept in the product market is assumed to be that of Bertrand-Nash. Solving for the product market equilibrium and substituting the relevant terms into (6), it can be verified (See Appendix A.1. for details) that the endogenously chosen costs are given by,

$$\gamma[\frac{\partial\beta}{\partial c_i}]_{\beta=1} + \frac{\lambda}{c-\alpha} = 0 \tag{8}$$

where β denotes the ratio $\frac{\Omega_i}{P_i}$ and γ is a constant. Partially differentiating the above expression with respect to n and imposing symmetry gives us $\frac{\partial[\frac{\partial\beta}{\partial c_i}]_{\beta=1}}{\partial n} < 0$ (See Appendix sections A.1. and A.2.). This establishes the main point to be taken away from this counter example: With a change in assumptions regarding product market competition, a result opposite to that of Martin (1993), confirming the popular intuition may be obtained - firm costs can be shown to be inversely related to the number of firms competing in the market.[8]

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[7] This demand function possesses some desirable properties relative to the familiar linear demand functions (favored in textbook treatments) which take the form, $Q_i = A + \Omega_i - P_i$. With such linear demand, if the difference between own price and average price charged by the rest of the market stays the same, demand stays the same - regardless of how high these prices are. With our specification of demand, even if the difference between own price and average price charged by the rest of the market stays the same, demand is lower as own price gets higher.

[8] It should be pointed out that the demand function we have chosen implies that entry has no effect on per-firm output (as can be seen directly from the fact that imposing symmetry in (8) gives us demand for i that is independent of the number of firms in the market). This is a convenient property (although not a necessary one) for the point we wish to make and is so for the following reason: As discussed earlier (immediately following equation (6) in the text), the change in the direct effect due to entry generally leads to *higher costs* being chosen by owners. For the opposite to obtain (as it does in our counter example) what one needs is for the change in the strategic effect to dominate the change in the direct effect. In general, demand functions that do not have per-firm output (i.e., the direct effect) drop too much due to entry are more likely to give us the result that greater competition

In deriving this contrary result, we have made two changes to the product market assumptions of Martin (1993). First, we have switched from the Cournot (strategic-substitutes) game assumed by Martin (1993) to a Bertrand (strategic complements) game. Second, we have assumed a different form of demand. It is natural to ask the question of whether it is the former or the latter change that is responsible for this drastic difference in model predictions. Unfortunately, this cannot be cleanly apportioned. To see why, recall (from the discussion following (6)) that it is the *change* in magnitude of the direct and strategic effects following entry that is central here. While strategic complements/substitutes help us pin down the sign of the strategic effect, *changes* in its magnitude are determined by both firm conjectures and properties of the demand function. This is true for the direct effect as well. Whether greater competition lowers costs or raises the sum (of the direct and the strategic effect) will therefore depend upon the specific assumptions of the framework under consideration and generalizations cannot be easily made.[9]

Appendix

A.1. Given (7) and our assumption that firms hold Bertrand conjectures, it is easily verified that the first order condition for the i^{th} firm's maximization problem is $P_i c_i = 2\theta c_i \Omega_i - P_i \theta \Omega_i, i = 1......n$. Given symmetry, any firm i recognizes that the other firms are all identical to one another and therefore recognizes that their optimal cost choices in the previous stage and prices chosen in the present stage are identical to each other as well. From any firm $i's$ standpoint, a solution to the (above) equations defining the reaction functions can be shown to take the form $\beta = \frac{\Omega_i}{P_i}$, where β is a function of own costs, c_i, the other firm's costs, c, and the number of rival firms n. Plugging these into the reactions functions gives us that the equilibrium price charged by firm i can be written as, $P_i = c_i f(\beta)$. This (along with some tedious algebra) gives us that β is defined implicitly by $\beta = (\frac{c}{c_i})[\frac{f(\frac{1/\beta+n-1}{n})}{f(\beta)}]$, where further $f(\beta) = \frac{2\theta\beta-1}{\theta\beta}$.[10]

increases efficiency since it is more likely then that the change in the direct effect will be dominated by the change in the strategic effect. This obtains to an extreme extent with our demand function - since per firm output does not change at all with entry.

[9] Thus, for instance, it is easily verified that in a (linear) Bertrand model with demand represented by $Q_i = \frac{1}{n+1}(A - P_i + \Omega_i)$, efficiency is lowered by entry. In this case, the final equation determining costs, analogous to (8), is given by $\frac{\lambda}{c-\alpha} = \frac{[(2+\frac{1}{n})A - \frac{1}{n}c]}{(n+1)(2+\frac{1}{n})^2}$. That entry raises the level of cost endogenously chosen is verified by noting that in the right hand side term, the numerator is decreasing in n and the denominator is increasing in n for all $n \geq 2$. Thus, simply switching to Bertrand conjectures does not guarantee that efficiency increases with entry. Equally, it may be verified that switching to non-linear demand in itself does not guarantee increased efficiency.

[10] Existence and uniqueness of a solution for β (for positive costs and prices) can be easily verified. Consider the right hand side expression defining β above, $G(\beta) = (\frac{c}{c_i})[\frac{f(\frac{1/\beta+n-1}{n})}{f(\beta)}]$. For given costs, we can see that this expression takes the limiting value of zero as β approaches zero. For all values of β between zero and $\frac{1}{2\theta}$, it takes on negative values (since the numerator is always positive while the denominator takes on negative values in this above range). Clearly, β cannot lie in this region.

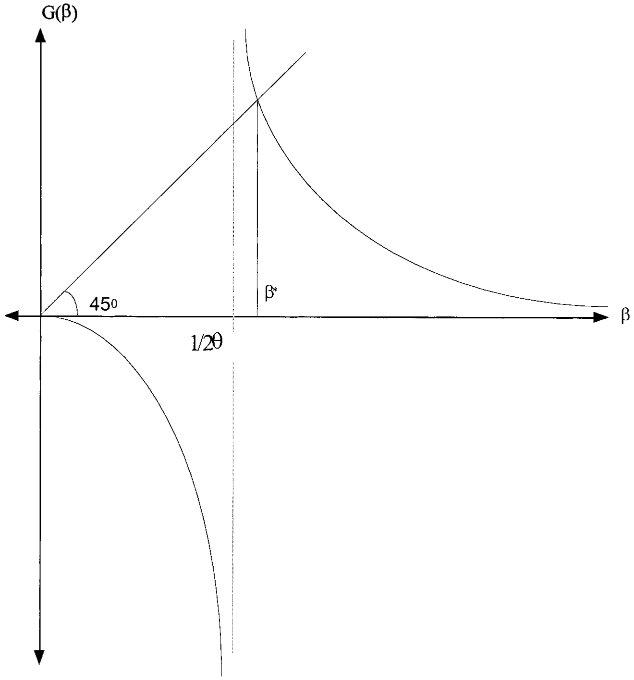

Figure 1

The modified reaction function above, $P_i = c_i f(\beta)$, also implies that firm $i's$ equilibrium profit level (excluding managerial compensation) can in turn be written as $\Pi_i = [\frac{(f(\beta)-1)(\theta\beta-1)}{f(\beta)}] = H(\beta)$. Using this in (5) and *then* imposing symmetry (noting that $\beta = 1$ implies and *is implied by* $c_i = c$) gives us (8), where $\gamma = H'(1)$.

A.2. Starting with β as implicitly defined in A.1., we have

$$[\frac{\partial\beta}{\partial c_i}]_{\beta=1} = -[\frac{f(1)}{c(f(1)+f'(1)(1+\frac{1}{n}))}].$$

It is easily verified from the definition of $f(\beta)$ (See also footnote 10) that all the terms inside the square brackets on the right hand side are positive . Thus, directly, $\frac{\partial[\frac{\partial\beta}{\partial c_i}]_{\beta=1}}{\partial n} < 0$.

Further, at values of β just greater than $\frac{1}{2\theta}$, the expression takes only positive values (approaching infinity as beta gets closer to $\frac{1}{2\theta}$). Given that the numerator is increasing in β and the denominator is decreasing in β, this expression is decreasing in β. It should be clear that, for given costs, there is a unique β and that it takes values $> \frac{1}{2\theta}$. Figure I illustrates.

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